GENERATING A MODEL OF THE TERRAIN
IN THE ASPECT OF FLOOD RISK ESTIMATION

GENEROWANIE MODELU TERENU W ASPEKCIE
SZACOWANIA RYZYKA POWODZIOWEGO

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Abstract. Flood modeling is carried out based on the Digital Terrain Model (DTM) and hydrological data. The quality of the input data for modeling determines the reliability of flood risk assessment. The accuracy of the DTM is affected by the method of data acquisition and mathematical modeling errors. This paper presents a method for evaluating the impact of the accuracy of measurement and modeling based on the DTM. The analysis was performed on a hypothetical example generated by statistical methods. This method can be used to analyze various aspects of precision in creating DTMs.

Streszczenie. Modelowanie zagrożenia powodziowego prowadzi się w oparciu o Cyfrowy Model Terenu (DTM) i dane hydrologiczne. Jakość danych wejściowych do modelowania determinuje niepewność oceny ryzyka powodziowego. Na dokładność DTM wpływ ma metoda pozyskania danych i błąd modelowania matematycznego. W niniejszej pracy przedstawiono metodę oceny wpływu dokładności danych pomiarowych i modelowania na DTM. Analizę przeprowadzono na hipotetycznym przykładzie generowanym metodami statystycznymi. Metodą tą można analizować różne aspekty dokładnościowe tworzenia DTM.

Key words: geodetic measurements, pseudorandom number generator, Digital Terrain Modeling, flood zones

Słowa kluczowe: pomiary geodezyjne, generator liczb pseudolosowych, Cyfrowy Model Terenu, strefy zalewowe

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INTRODUCTION

Intense precipitation causes an increase in the level of water in watercourses, which with reduced water absorption capacity of the soil may result in flooding. The flash flood arising as a result of heavy rainfall or a violent rainstorm can transform a small watercourse into an element, destroying everything in its path [Michalec and Tarnawski 2006, Bartnik and Książek 2010].

In terms of reducing risks to people and property due to flooding, adequate flood prevention is of key importance. One element of this prevention is to determine flood zones adjacent to watercourses. The Act requires the preparation of anti-flood protection studies in which immediate and potential risk areas must be indicated, as well as areas requiring protection because their economic or cultural value, and type of development. The Floods Directive resulted in the need to develop flood hazard maps and flood risk maps. These maps are based on hydrological data and spatial data on the topography of the terrain.

Determining flood risk zones. This involves the generation of a Digital Terrain Model (DTM), which is a reflection of the actual shape of the terrain with a Digital Water Surface Model (DWSM), providing information on the elevation of the flood water level.

DTM is created based on data from geodetic measurements processed using suitable mathematical algorithms. Creating a DTM based on extensive surveying is burdened with all kinds of errors. Measurement errors and development of the results may significantly affect the quality of the resulting land surface model [Hejmanowska 2013]. In this work the issue of creating the DTM based on geodetic measurements shall be presented in reference to the creation of flood hazard zones. Also an attempt to determine the accuracy of the DTM shall be presented, within the valley of the watercourse. As a criterion for assessing the accuracy of the assumed capacity of the watercourse valley. The calculations carried out shall be made on a hypothetical example using statistical methods.

THE ROLE OF DTM IN DETERMINING FLOOD RISK ZONES

The flood zone, also called the flood plain, covers the expected reach of overflowing waters calculated based on historical or hypothetical data, with some probability outrun [Ciepielowski 1999]. Modeling flood zones facilitates decisions on countering the effects of flooding. In the proximity of water courses where there is a possibility of flooding, an error in a digital terrain model can affect to an extreme extent the importance of the evaluation of risks to life and property of the population.

One of the essential elements of rational water management is the modeling of flood zones [Nachlik et al. 2000, Radczuk et al. 2001, Muncaster et al. 2006, Książek et al. 2010]. In many countries, information on the borders of flood zones is widely available. Therefore, the relevant state institutions create maps with the areas where there is some risk of flooding [Hejmanowska 2005]. These maps are used for the analysis of flood risk and are used to create scenarios of flood, which allows the relevant institutions to make decisions faster in case of a flood [Depczyński and Szamowski 1997]. This is highly important because it affects short response time to minimize risks and to reduce the effects of floods [Słota 1999, Radecki-Pawlik 2010].
Designating flood zones, there are two possible cases of the risk of flooding. The first case concerns natural occurrences of a given level of water. Then, one shall also rely on historical information regarding the amount of water in a particular year, which is often placed on buildings. In such a situation, it is very easy to determine, on the basis of an altitude map, which areas were flooded. Knowing the flood water level, one can determine the areas with an elevation smaller than that contour. The second case relates to flood plains which may arise, for example, as a result of damage to hydraulic structures, such as levees or weir dams [Michalec and Tarnawski 2007].

The primary resource of information used for creating models of flood zones is the Geographic Information System. Data on the topography are acquired by means of geodetic methods. These surveying methods allow one to achieve higher accuracy of input data for modeling terrain. The technology used in measuring, the point density, along with terrain and environmental conditions may, however, cause errors of input data for the modeling.

Creating a digital model of the terrain surface is conditioned by many factors, which may adversely affect the accuracy of the resulting model and its deviation from the actual shape of the terrain. The choice of the mathematical model, the adoption and implementation of its parameters and the calculation can cause significant distortions. Adding measurement errors to that, this can result in unsatisfactory effects. Significant errors in a DTM can bring erroneous conclusions regarding flood risks in an area.

The most important element of the Geographic Information System (GIS) is a DTM. It is the DTM that is used to create models of flood zones. When there is no DTM for an area, or its accuracy is unsatisfactory, land surveying should be carried out and only then one can generate the model. The basic problem associated with the use of DTM is that formed with different accuracy within the same area, due to the different methods of data acquisition. These differences may even exceed 1 meter. And this, in turn, has a large impact on the accuracy of modeling flood zones [Hejmanowska 2005]. Therefore, it is important to accumulate not only data but also information on the accuracy of the input data for modeling a DTM.

**DETERMINING FLOOD RISK ZONES**

There are various possibilities to determine flood risk zones. The hard method involves the determination of trace intersection of the DTM with the water table elevations. The soft method involves taking into account the imprecision of input data in the analysis and determining the likelihood of a flood zone [Hejmanowska 2005].

If we take into account the inaccuracy of the data, the soft method should be used in the analysis. It determines the probability with which a given area will be flooded, i.e. assuming a certain DTM error and accuracy of the hydraulic model, the probability of exceeding the preset values in the difference map is determined. By subtracting the values of elevation of points from DTM values for the water surface level, a differential map is acquired where points with a difference in value with the sign “−” are flood areas, and those with a “+” non-flood areas. Points with a zero value difference form the borders of the flood zone.
The “soft” method allows one to determine the flood zone boundaries with a certain degree of risk (for the assumed confidence level) that other areas, outside the specified zone, may also be flooded. Assuming that the errors are normally distributed (which is not always true), have the expected value $\mu$ and standard deviation, $\sigma$ the density function of the normal distribution is:

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

(1)

Assuming that in the case of generating flood zones, the expected value is 0 m, and the standard deviation is $\pm$ 1 m, the distribution function of this distribution will be:

$$P(X \leq x) = F(x, 0, 1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{x^2}{2}\right) dx$$

(2)

Figure 1 shows a graph of the distribution function resulting from the formula (2).

Fig. 1. Cumulative distribution function of model error

Ryc. 1. Dystrybuanta błędu modelu

Analyzing the distribution function chart (Fig. 1), the values of error are converted into the probability of error, so that in the difference map it will be smaller than the error of the flood zone boundaries. For example, to acquire a 5% risk (95% probability), one shall take into account only those points from the differential map which have values of less than 1.64 m.

The end result of the soft method calculation shall be a probability map representing areas with less than zero values on the difference map.
THEORETICAL EXAMPLE

The evaluation of the effect of measurement conditions and modeling on the quality of the DTM is difficult to carry out on the basis of actual results of measurements carried out in reality, due to the lack of a corresponding set of repeatable measurement results. For this reason, it is convenient to use numerical methods, which are quite flexible and allow the acquisition of results of a qualitative nature. Generating multiple terrain models using a random number generator allows one to analyze the impact of various factors on the modeling. For quantitative assessment of the modeling error a hypothetical calculation of the volume changes of the river valley was assumed, depending on the water level changes.

For the calculation one can assume a hypothetical equation for the watercourse valley. Using a random number generator, “measurement results” for a hypothetical surface can be created, corresponding to multiple series of measurements.

To carry out the theoretical considerations one can assume that the valley can be described by an equation of the surface with a cross-section similar to a Gaussian curve. The following theoretical equation of the surface can be proposed:

\[
\begin{align*}
-20 &\leq x \leq 20 \\
50 &\leq y \leq 150 \\
z &= z_0 - \frac{k^3}{k^2 + x^2}
\end{align*}
\]

(3)

where \( z_0 \) and \( k \) are parameters that control the shape of the cross section.

For the simulation of measurements, the following values of these parameters were assumed:

\[
\begin{align*}
z_0 &= 282.5 \\
k &= 10.0
\end{align*}
\]

(4)

In the equation (3) the dimensions of 100 × 40 meters for the valley were adopted. The section through the surface (3) with parameters (4) can be represented as in Figure 2.

For such a surface, by means of the adopted random number generator, theoretical “measurement results” of the surface coordinates are generated. We shall assume that the measurements were carried out in a square grid 5 × 5 m. The accuracy of points’ placement in the horizontal plane is ±1.0 m, while the “measurement” error in the vertical plane is ±0.20 m. The analysis in this study was conducted in two case variants named A and B. For each of the cases, “measurement results” were generated using a random number generator, for the number of 300 samples. In the case of A, assumed values were generated exclusively outside the hypothetical watercourse; therefore, the model has no values within about 0 meters from the watercourse axis, while in the case B the results included the axis of the points in the region of the watercourse. Figure 3 shows the arrangement of dots generated in both cases.
Figure 3 shows the area analyzed in the inner line. As can be seen, the points were generated outside the analyzed area, in order to avoid distortion on the edges.

A sample cross section of the surface for generated realization in the area of 95 meters is shown in Figure 4. In this Figure, the black line indicates theoretical values in the profile, and groups of dots represents generated points.
To generate the digital terrain model and volume calculations, the program Surfer from Golden Software was used [Yang et al. 2004, Erdogan 2009, Litwin et al. 2013]. The terrain model was created as a grid of squares measuring 5 × 5 meters by the means of the Kriging interpolation algorithm [Grzelak and Kwinta 2013]. On the basis of the generated mesh squares, using simple geometric solids (extended trapezoidal method) the volume of the hypothetical watercourse was calculated up to the level of the water surface from 273 to 280.5 at intervals of every half a meter.

In order to compare the results of calculations for the generated surface with strict values corresponding theoretical calculations were carried out. Symbols as in Figure 5.

In accordance with the formula (3) and symbols as in Figure 5, the volume depending on the water surface can be described by the formula:

\[ V(z_w) = \left[ 2z_w x_w - \int_{-x_w}^{x_w} \left( z_0 - \frac{k^3}{k^2 + x^2} \right) dx \right] (y_{\max} - y_{\min}) \]  \hspace{1cm} (5)
After the transformations and integration the following is obtained:

\[ V(z_w) = 2 \left[ (z_w - z_0)x_w + k^2 \arctan \frac{x_w}{k} \right] \left( y_{\text{max}} - y_{\text{min}} \right) \] (6)

The distance value within a given water level as a function of the surface level \( x_w \) results from the transformation of the equation (3) to the following form:

\[ x_w = \pm k \sqrt{\frac{k}{z_0 - z_w}} - 1 \] (7)

The results of the calculations are presented in the two tables below. Table 1 refers to the results of simulation for the case A, while Table 2 presents the results of calculations for the case B. In the following columns, the tables summarize the elevation of the water table, the distance from the axis of the valley (7), the theoretical volume (6), the maximum volume generated, and then the minimum and average volume. The next column lists the mean error and limiting error (3 times the average error), and on their relative values (normalized to the theoretical volume).

Figure 6 shows the results of calculations in graphical form. We compared the average volume results obtained from the two cases analyzed with the strict theoretical result. As can be seen from Figure 6 and Tables 1 and 2, that the differences in volume caused by assumed geometric errors in the data used for modeling the DTM are noticeable, while at the level of the obtained values they are not significant.
Table 1. Calculation results of case A
Tabela 1. Wyniki obliczeń dla przypadku A

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Table 2. Calculation results of case B
Tabela 2. Wyniki obliczeń dla przypadku B

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<td>14082.1</td>
<td>36.5</td>
<td>109.4</td>
<td>0.3</td>
<td>0.8</td>
</tr>
</tbody>
</table>
CONCLUSION

The hard method, which generates only the boundaries of flood zones, is still the one of choice in modeling flood zones. To ensure the safety of people living in the vicinity of watercourses, the soft method of modeling flood zones should be used. The latter method, apart from generating a trace of the intersection of the DTM with the water table level allows one to account for the risks associated with inaccurate source data. Knowing the analysis error, one can establish the cumulative distribution function, and thus calculate the probability for particular points, which can have a positive impact on the quality of the image obtained of the flood risk.

The process of creating the DTM may be affected by numerous errors, which may result in drawing erroneous conclusions regarding modeling flood zones. The method to estimate the error of creating the DTM, based on a random number generator, presented in this paper can be widely used to assess the impact of errors on the result of modeling. This way, one can analyze, for example, the measurement error, the size and distribution of measurement points and the choice of methods for creating DTMs from input data. The results of modeling errors can be introduced to the soft method of generating flood risk zones.

Fig. 6. Comparison of the theoretical volume with the average values for the calculation cases in the function of water altitude

Ryc. 6. Porównanie objętości teoretycznej z wartościami średnimi dla przypadków obliczeniowych
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